AEROSPACE LABORATORY

GENERAL INFORMATION MANUAL

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1 Introduction

Experimental work, in most situations you will encounter, is quick and dirty compared to the systems you will find in the Undergraduate Laboratory. Typically, experimental setups are assembled from parts of others to complete a specific task (usually a graduate degree) and really only function for about 5% of the total time a researcher spends creating it and debugging it. Connections are nearly always made in a temporary manner from borrowed cables and alligator leads. A week after the project is complete and all the lenders have reclaimed their pieces, the apparatus sits as a giant pile of equipment, not a coherent system. The next researcher along invariably moans that nothing ever works and they can't believe the results the last researcher got.

Computer data acquisition is used only where the volumes of data exceed a human's ability to record it. Data acquisition systems will invariably write the data to a proprietary file format on an isolated computer system requiring extensive effort to get data to an analysis program on a workstation or central computer. Of course, the analysis program, if one exists, never does what is required for this experiment and was written 3 researchers ago. Months, sometimes years are spent doing an experiment that actual takes less than 24 hours to perform.

Experimental work is hard. It is managing many small variables to produce acceptable results. Experimentation is a skill that a researcher must develop. A large part of the experimentation is learning patience and learning to do things correctly the first time. Every task must be done in a careful, methodical manner and then, no matter how simple the task, it must be rechecked at least once to guard against error. A good experimentalist knows the limitations of budgets and circumstances. Given these constraints, they will assemble the system with the greatest accuracy and precision possible.

2 Laboratory Format

The laboratory incorporates 4 experiments; 1 for Mechanics of Solids and Structures, AER373F; 2 for Aerodynamics, AER307F, and 1 for Gasdynamics, AER310S. Preparations must be handed in on the hour at the start of the laboratory. Formal reports must be handed in one week after the experiment is performed. The laboratory notebook entry for each laboratory experiment will be marked after the report is handed in and before the end of the term.

The website associated with the laboratories is: www.courses.ece.utoronto.ca/aer300y/. Any required files may be found here.

2.1 Preparations

Your report preparation answers must be submitted at the beginning of the class before starting the experiment. Prepare your answers on loose leaf paper, these will not be returned. A mark of zero will be assigned if a preparation is not submitted promptly at the beginning of the lab period.

2.2 Reports

Reports must be handed in to the instructor in a blue Faculty report cover (available from the Engineering Stores) no later than one week after completing the experiment. Reports will not be returned.

Penalty marks will be assigned to late reports at the rate of 5% per business day. The Undergraduate Lab drop box is available for after hours and weekend drops. Students are asked to write the date and time that the report was dropped off on the report before depositing it – if it is absent, the day and time it is removed from the drop box will be taken. The end of one submission day and the beginning of the next will be taken as until the instructor leaves on lab days and 5pm on non-lab days. This honour system will continue until there is cause to believe it is being abused.

3 Recommended Guidelines for Experiment Reports

The text portion of a report (from Introduction to Conclusion) must not exceed 10 double spaced typed pages but a good report will be shorter. Marks will be deducted for oversized texts. There is no restriction on the number of Figures, Tables, Computer printouts and Programs. Marks will NOT be deducted for a handwritten report. If using a word processor, you may use it to format equations and symbols only if it does them properly, otherwise it is preferable and more expedient to write them in neatly by hand.

After years of marking reports, it has become clear the people who produce the longest text sections in their reports usually have less of an idea of what they wish to convey and invariably end up conveying less. In producing monster reports students do not increase their mark, they waste tremendous amounts of time, usually in front of a computer, and then complain the reports require an undue amount of work. Since the reports are general worth more than problem sets in other courses, students are tempted to spend every hour between doing the experiment and handing in the report working on the report to the detriment of their other courses. It is strongly recommended that you do not spend more than 8 to 10 hours preparing any of these lab reports.

The suggestions provided here serve merely as pointers for the production of a good report. There is no absolutely correct methodology for report preparation - a critical reviewer can always find a few i's to dot or t's to cross. Style manuals which set the standard for scientific journals change with time, and this is good; nothing should be cast in stone, unless it really is the last word. Most students will be submitting papers for publication in the near future and reports are an excellent vehicle for honing formatting and literary skills.

(a) Be logical, consistent, clear and concise in your work. Guide the reader through the steps you have covered during the experiment. Write only to convey information. Think of each sentence as a tool, each paragraph as a component of the larger work. Do not meander; this makes marking very difficult. Use tables and figures liberally, use words only where necessary.

(b) Try to be neat, employ good English (check your grammar and spelling), don't invent words. In keeping with faculty policy to increase the communications skills of Engineering students, marks will be deducted and suggestions will be provided.

(c) Do not rewrite ANY text from the lab manuals into your report. You may use equations from the manuals and photocopy figures for inclusion in your report.

Table of Contents

Please include a table of contents – it provides a useful guide, and only the page numbers change from report to report. Here is an example table of contents with estimated page lengths for each section. Reports will not be marked on how well their sections conform to these estimations.

		Pages
1	Introduction	< 1/2
2	Notation	1
3	Experimental Procedures	1 line
	and Results	1 page
4	Error analysis	2
5	Discussion	2 - 4
6	Suggestions	< 1/2
7	Conclusions	< 1/2
8	References and Bibliography	
	Tables	These are your results
	Figures	These are your results
	Appendices	

3.1 Introduction

Give the reader a concise overview of the experiment and its significance in the aerodynamic field.

3.2 Notation

Photocopy, cut and paste this page from the laboratory manual and add additional entries to the bottom of the list.

3.3 Experimental Procedures and Results

Do not rewrite the Experiment Procedure from the lab manual in your report; it is sufficient to record: "The procedure from the laboratory manual [1] was followed." – one line and one reference. State any significant deviations from the procedures described. To save time, save a model word processor file with the report section headings and this written under Procedures. Have the laboratory manual as Reference [1] under References. Duplicate this file to start each new report.

Refer to all the results you have summarized in the tables and figures. Guide the reader, explain what you did, refer to sample calculations, computer programs and sample outputs in the appendices — organize and present your results in a concise coherent fashion. Make simple observations based on the results: "The relationship is linear." Refer to equations where needed — rewrite them from the lab manual if required to clarify your descriptions. Include photocopies of important figures if necessary. Always provide sample calculations in an appendix. If you have some analysis of your own, include it in this section. This section is the glue that makes all of the tables and figures in the back of the report make sense — use only as much glue as is necessary to get the reader to the next section.

3.4 Error Analysis

This section is very important not only for the numerical reliability you can attach to your results, it also serves as a measure for evaluating the sources of error associated with the equipment and the methods used. Review your data critically. Refer to sample error calculations in the appendices. Reading errors should of course appear in the tables that summarize your raw data. Do not say anything more about reading errors associated with reading a static scale. Give estimates for the error bounds on your final results (these must be accompanied by sufficient calculations, etc., to give them credence). Comment on the sources of error in the experiment, quantifying which are large errors and which are small errors.

Assessing the error in an experimental measurement requires skill and experience and each lab experiment should be used as an opportunity to develop those skills. Read the section on "The Collection and Treatment of Experimental Data" in this manual, and adhere to those precepts.

3.5 Discussion

It is difficult to define what must be in a discussion. Certainly, students should discuss errors, not how they numerically affected the results — that is the Error Analysis section — but perhaps how the method or principle of measurement produces them. Such analyses could easily lead to suggestions of possible modifications to the method or apparatus but actual suggestions should be made in the Suggestions section.

Certainly something should be said about the results, but don't rehash the Results section — say things that are not simple observations, things that illuminate the significance of the results. What did the experiment prove?

Certainly there should be some evidence of external reading in this section. Do not cite books merely to bring in irrelevant facts or to support material already in the laboratory manual. Use external reading to say something new.

Answering some of these questions could be part of the discussion: Could analysis alone have provided the answer without the equipment? Has the experiment been well designed for tutorial purposes? Was the equipment adequate for the task? Could the apparatus be employed to measure some other useful parameter which would enhance the experiment (maybe with a small modification)? Do your results agree with the theory? If not, offer explanations. Can you extrapolate your data to reach toward further conclusions? Comment on trends and deviations, etc.

Be critical, however, a diatribe of complaint is inconsequential unless accompanied by suggestions for viable improvements. For example, a sticky meter or a leaky valve is not a major problem and deserves at most one line of comment – particularly when following common sense procedures completely alleviates the problem.

3.6 Suggestions

Suggest improvements or changes to the method or apparatus.

3.7 Conclusions

This section should be concise, summarizing the salient features of your work; it should be no more than one page in length. What did you discover from your work? Was agreement with theory good? Did your discussion lead to any conclusions?

3.8 References and Bibliography

References are the listing of the bibliographic data for books actually referenced in the report. Reference [1] will likely be the laboratory manual as cited in the Experimental Procedures section. Entries must be listed in the order they are referenced.

The bibliography contains a listing of all other books read that contributed to the report but were not explicitly referenced anywhere in the text. The bibliography entries must be listed in the alphabetical order by the first author's last name. References always precede bibliography entries.

3.9 Tables

Every piece of raw data you collected should appear in a table with a reading error quoted. All intermediate answers (if any) should also be summarized in tables. All final answers from a series of runs of the same type should be summarized in one table. This summary is useful for comparing the different samples or showing the affect of a varying parameter. Tables need not be elegant tables produced by word processors. A formatted output from a computer program, with proper headings and notations neatly added, could make a fine table. Tables always proceed the figures.

3.10 Figures

In some contract reports, engineers are paid by the figure or graph because this is where the real technical information is communicated. A large part of the text is only to bind the report together and explain the data in the tables and figures.

Graphs should present every set of data where there are trends not adequately seen in a table — like a linear relation or some other curve. Experimental data should always be plotted against a theoretical curve if one exists. Graphs should always include error bars, a title: "Table 1: Displacement vs Time for a 5m Diameter Rolling Wheel", axis labels: "Displacement (x) in meters", a grid and, if hand drawn on graph paper, a logical ascendance of axis scale with grid lines. For instance, do not mark units of 5m for every 7 squares of graph paper, use 5 or 10 squares.

DO NOT use a computer plotting package to produce microscopic, inferior plots. Matlab does an acceptable job (subplot(111)) and has the ability to plot with error bars and a usable grid. Please draw a smooth line on the paper using French curves if your graphing package does not do a good job. Graphs that are small or lack a useful grid or error bars may be used to compare data from two or more sources or used to show trends. With the proper presentation of experimental data, a reader can easily and accurately extract technical data from your graphs — the purpose of a graph in a experimental formal report. Journal papers use microscopic graphs with no grid because no one is expected to read data from them. A report with hand written equations and full page, hand drawn plots on proper graph paper, with error bars where appropriate, is superior to most computer generated material. Hand produced material in these areas is acceptable to any contractor who is interested in the results of the experiments.

Diagrams should be inserted, photocopied from the lab manual or any other source (properly referenced) where necessary to guide the reader. If you are trying to explain an idea of your own, instead of spending two pages of text explaining it, sketch two diagrams and spend less that half of a page on the idea.

Do not intersperse 8.5'x11' figures with your text. From time to time, photo reducing a diagram and inserting it into the text is desirable. Do not photo reduce a graph unless it is only to show trends as in scientific journals.

3.11 Appendices

This is the section for all the data, printouts, computer programs, Matlab scripts and sample calculations Without this material, the report would be incomplete but it is too detailed and bulky to place in any other section.

4 Laboratory Notebooks

Each experiment has some information or data which will need to be recorded. This should be done in a hard-cover laboratory notebook. These are the permanent records of your experimental work. Remember that in the industrial milieu a lab record may provide temporal confirmation for patent assignation, so be sure to date all entries. All lab books will be marked before the conclusion of the term.

Some notebook guidelines are listed below:

(a) Put your name on the front cover.
(b) Reserve the first two pages for a Table of Contents.
(c) Number each page, Date each page
(d) Don't tear out pages - just cross out any irrelevant entry.
(e) Always use pen! Pencil is not permanent.
(f) Each student should keep his/her own complete data record for all experiments. If your partner records the data be sure to obtain your own copy. Data entered directly into laptop computers (etc.) should be printed out and attached in the notebook.
(g) Write down everything you do - you may forget something by report writing time.
(h) Note in particular any deviations from the lab manual procedures.

5 Report Marking Procedures

The marking scheme employed is given below and the approximate mark for each section is listed in parentheses, with the following caveat: The amount of data reduction, analysis, error estimation, number of figures, tables, etc., varies for each experiment and the weighting for each of the elements listed is adjusted slightly to reflect their relative contributions.

(a) Introduction (3): suitable problem summary and preamble

(b) Notation (2): consistency and correct values for constants

(d) Experimental procedures and results (20): sample calculation and independent analysis, data presented in the tables and figures

(e) Errors (15): for the formal reports your error estimation should contain a correct numerical evaluation and consideration and discussion of all sources of error

(f) Discussion of results (20): comprehensive assessment and interpretation of all aspects of the experiment, comparison with theory, evidence of external reading, etc.

(g) Conclusion (5): concise and pertinent summary of the experiment and results

(h) References, Bibliography (5)

(i) Appendices (5): should be reserved for analysis or auxiliary data germane to the report but too length to include in the main text

(j) Tables (5): titled headings, logical layout, all necessary parameters listed, units/dimensions, error bounds The presentation of the tables will be marked here. The data contained was already marked under (d)

(k) Figures (5): titled captions, photocopies where necessary, graphs should have legends, labels on axes, curve fits with equations where appropriate; give equations for your fits and correlation coefficients, units/dimensions The presentation of the figures will be marked here. The data contained was already marked under (d)

(l) English (5): coherence, sentence structure, grammar, spelling. If the use of language is so confusing the technical information is not communicated, marks cannot be granted for the material that was not communicated.

(m) Legibility and neatness (5) - usually an issue only in sample calculations and notations on figures. Handwritten reports may suffer in this category if the writing is very difficult to read. If it is not legible at all, marks cannot be granted for the material that was not communicated.

(n) Original ideas and suggestions (5): Use a separate section for this category

6 Course Mark Breakdown

Course marks are based on written formal reports, answers to preparation questions, and the laboratory notebook entry for the experiment. Each experiment is worth 7.5% in AER307F and 10% in AER373F and AER310S.

Preparations	20
Reports	100
Lab notebook	5
Total	125

7 Reference Texts Available

The following texts are available on request. All books are reference only for use in the Aerospace Laboratory. All texts referenced in your laboratory manuals are available.

Fluid Dynamic Drag, S. F. Hoerner
Catalog of Low Reynolds Number Airfoil Data for Wind Tunnel Applications, S. J. Miley
Theory of Wing Sections I. H. Abbott, A. E. Von Doenhoff
Applied Aerodynamics, L. Bairstow
Airplane Performance Stability and Control, C. D. Perkins, R. E. Hage
Aerodynamics of V/STOL, Flight, B. W. McCormick
Introduction to Flight, J. D. Anderson
Fundamentals of Aerodynamics, J. D. Anderson
Foundations of Aerodynamics (2 copies), A. M. Keuthe, J. D. Schetzer
Wind Tunnel Testing, A. Pope
Low Speed Wind Tunnel Testing, A. Pope
High Speed Wind Tunnel Testing, A. Pope, K. L. Goin
Applied Hydro and Aeromechanics (2 copies), L. Prandtl, O. G. Tietjens
Fundamentals of Hydro and Aeromechanics, L. Prandtl, O. G. Tietjens
Boundary Layer Theory, H. Schlichting
Principles and Practices of Laser Doppler Anemometry, F. Durst, A. Melling
The Laser Doppler Technique, L. E. Drain
Elementary Mechanics of Fluids, Hunter Rouse
Fluid Dynamics, V. L. Streeter
The Dynamics and Thermodynamics of Compressible Fluid Flow (Vols. 1 and 2), A. H. Shapiro
Elements of Thermodynamics and Heat Transfer, E. F. Obert

Elements of Gasdynamics, H. W. Liepmann, A. Roshko
Handbook of Tables for Applied Engineering Science, CRC Press
Handbook of Chemistry and Physics, CRC Press
Handbook of Mathematics, CRC Press
Introduction to the Kinetic Theory of Gas Flows, G. N. Patterson

8 Collection and Treatment of Experimental Data

Some simple guidelines are given here to enable you to perform an accurate and succinct estimate of the errors in calculated parameters derived from your experimental results. While by no means comprehensive the information given here is adequate for all the Aerolab experiments. Sometimes the error analyses handed in are either inadequate, far too extensive (and sometimes wrong) or irrelevant.

8.1 Collection of Data

(a) Note any idiosyncrasies associated with the equipment. For example, if there is a ± 1 mm deviation of the zero value on a manometer board with no pressure input, then it is pointless to record data to 0.02 mm. Similarly, if flow fluctuations cause the column height to vary by ± 2 mm, make a note; this could be the most significant error in your results. Such variations may also change with different flow conditions so do not automatically assume a constant error throughout a series of observations.

(b) Do the readings drift with time?

(c) If you repeated a set of readings would you obtain the same values?

How would they change if you repeated your observations several hours after the equipment had remained unused?

(d) Remember **precision** is NOT the same as **accuracy**. It is possible, for example, to use a 5 digit voltmeter and measure a potential of 3.0246V but because of an instrumental defect the true value may be 3.2106V, i.e., 3.2V would be a more accurate value for the potential but 3.0246V is more precise (more significant figures).

(e) If you measure something in inches, pounds, etc., in the majority of circumstances you should convert to SI units for your analysis and presentations.

(f) Note all deviations from ‘normal’ (i.e., lab manual) procedures - be a detective and remain alert to observe any inconsistency or unusual behavior.

8.2 Treatment of Data

(a) Very few of the quantities measured in the laboratory will have an accuracy better than about 2%; however if you are using some unmeasured constants in your calculations do not round off until after you have finished your computations, i.e.,

ATMOSPHERIC PRESSURE	= 101.325 NOT 101 kPa (error = .3%)
AIR DENSITY	= 1.225 NOT 1.2 kg/m^3 (error = 2%)

It is easy to carry the extra digits through your computations and round off later - the two items above result in a 2.3% uncertainty and this will add unnecessarily to your experimental errors.

(b) Do ignore inconsequential errors. You should examine the measurements you are attempting to quantify - note which parameters are most significant and expend extra effort in improving their accuracy.

For example, if a quantity is squared in your analysis it will have its relative error doubled; therefore (if you can), make a special effort to measure it as carefully as possible.

(c) For almost all practical purposes you can assume a series of measurements on the same parameter is normally distributed (purists may shudder at this assumption but it is valid in most cases). Calculate the standard deviation - this requires a minimum of about 10 readings for a reliable estimate.

8.3 Error Analysis

(a) There may be occasions when you have no recourse but to carry deviations through your calculations in order to determine the error bounds in the final result. This is a powerful and practical method, however for the lab try to employ the analytical methods described below where possible; you will gain experience with the techniques and in most cases reduce the amount of work required.

(b) In general always quote the relative error (i.e., Re $\pm$ 5%); this can be appreciated at a glance - the reader knows that an error of 5% in a measurement of the Reynolds number is reasonable and although Re = 223197 $\pm$ 11160 is equivalent it is a little harder to discern the experimental accuracy.

Use the 'TOTAL DIFFERENTIAL' technique for the computation of errors where possible. You have probably all used this at some time or other but a few examples are given here to refresh your memory.

<u>EXAMPLE 1</u>

$$I = \frac{E}{R} \tag{1}$$

$$dI = \frac{\partial I}{\partial E}dE + \frac{\partial I}{\partial R}dR = \frac{1}{R}dE - \frac{E}{R^2}dR \tag{2}$$

$$\frac{dI}{I} = \frac{dE}{E} - \frac{dR}{R} \tag{3}$$

So the maximum relative error in I is

$$\frac{\Delta I}{I} \approx |\frac{\Delta E}{E}| + |\frac{\Delta R}{R}| \tag{4}$$

and if E and R are each measured with a tolerance of 1% then $\frac{\Delta I}{I} \approx$ 2%.

EXAMPLE 2

The relation $P = \frac{M\gamma^{\frac{1}{4}}}{D}$ can be treated as in Example 1 or:

$$\begin{aligned} \ln P &= \ln M + \tfrac{1}{4}\ln\gamma - \ln D && (5)\\ \frac{dP}{P} &= \tfrac{dM}{M} + \tfrac{1}{4}\tfrac{d\gamma}{\gamma} - \tfrac{dD}{D} && (6)\\ \frac{\Delta P}{P} &= \tfrac{\Delta M}{M} + \tfrac{1}{4}\tfrac{\Delta\gamma}{\gamma} + \tfrac{\Delta D}{D} && (7) \end{aligned}$$

The relative error contribution due to γ in this example is reduced by a factor of 4.

EXAMPLE 3

$$P = \frac{E^2}{R} \rightarrow \frac{\Delta P}{P} \approx 2\frac{\Delta E}{E} + \frac{\Delta R}{R} \tag{8}$$

Notice that in this example the relative error contribution of E is doubled, although some past reports have analyses which are equivalent to:

$$\frac{\Delta P}{P} = \sqrt{\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta R}{R}\right)^2} \tag{9}$$

which is simply wrong! Or:

$$\frac{\Delta P}{P} = \sqrt{2\left(\frac{\Delta E}{E}\right)^2 + \left(\frac{\Delta R}{R}\right)^2} \tag{10}$$

which is closer but still incorrect unless there is some a priori reason to reduce the total error by using the RMS mean (e.g., your deviations are 'most probable' errors) and the variables are independent.

Note that (8) requires less computation and gives the correct error bound for variables that may be non-independent.

It is interesting to note that historically some of the very best experimentalists have tended to underestimate the errors in their observations, sometimes quoting relative errors 2 or more times less than the true values.

You can extend the methods just described to embrace fairly complicated expressions quite easily and be sure you are not erring on the wrong side.

8.4 Estimation and Computation of Errors

A justification for the use of the error analysis techniques which should be used for all Aerospace laboratory reports is given here. Some students persist in using the RMS error approach, and unless this is used with an understanding of its shortcomings then the results must be considered questionable at best.

Consider the following example where it is desired to calculate the error in z, due to variations in the observations made on x and y, when $z = xy$.

RMS METHOD:

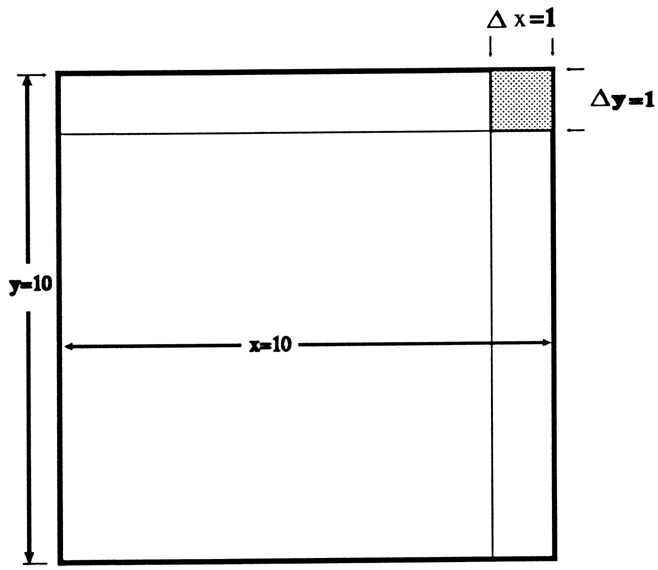
$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

AEROSPACE METHOD:

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

The latter clearly involves less computation and for more difficult expressions is simpler to apply when obtaining the basic relative error equation from a given functional relation.

The problem may be expressed geometrically and illustrated with some selected numerical values for x and y.



By direct substitution if maximum and minimum values into $z = xy$ we obtain:

$$z = 100, \qquad z_{min} = 81, \qquad z_{max} = 121$$

Therefore:

$$\Delta z = 19 \quad \text{or} \quad 21, \qquad \text{i.e.: } \bar{\Delta} z = 20$$

This may be contrasted with the RMS and Aerospace procedures which give:

<u>RMS METHOD:</u>

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} = \sqrt{0.01 + 0.01} \rightarrow \Delta z = 14.1$$

<u>AEROSPACE METHOD:</u>

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y} = \frac{1}{10} + \frac{1}{10} \rightarrow \Delta z = 20$$

In this specific example the Aerospace approach is closer to that obtained using the lower or upper bounds.

8.5 Some Points Often Neglected When Using the RMS Approach

When a statistical evaluation of errors is performed using the RMS method, the variables (x and y in our example) are assumed to be independent. This implies that when x is measured it does not influence the reading of y and vice versa.

If measurements are truly independent the associated deviations (errors) do not add vectorially because there is the possibility of compensation; i.e., if a high reading is obtained for x it may be accompanied by a low value for y. This adjustment mechanism can be compared to a random walk where N random steps of length L will result in a total distance traveled of $L\sqrt{N}$, instead of LN. The net effect is to produce a lower total error in z than would be obtained by using the upper and lower bounds.

Unfortunately independence between variables is seldom checked by experimenters although the calculation is simple (at least for our two-variable example).

$$\rho_{xy} = \frac{1}{N}\frac{\sum(x-\bar{x})(y-\bar{y})}{\sigma_x\sigma_y}, \qquad \sigma_x = \sqrt{\frac{\sum(x-\bar{x})}{N}}$$

The correlation coefficient ρ_{xy} must be zero to use the RMS method with equanimity, although in many situations the correlation between variables may be less than unity. In practice x and y are rarely truly independent and in some cases a careful examination reveals a degree of correlation between presumed independent variables that is surprising.

The Bottom Line

Perhaps the most compelling evidence for following the procedures recommended here is provided by the following table which lists measurements on the velocity of light obtained by highly competent experimenters.

The importance of a correct evaluation of 'c' has obvious ramifications throughout science, nevertheless history shows this field to be replete with examples of error underestimation.

Example: Velocity of Light in Vacuo

Author	Velocity (km/s)	Error Bound OK?
Michelson (1930)	299,796 $\pm$ 4	Y
Pease (1932)	299,774 $\pm$ 3	N
Birge (1941)	299,776 $\pm$ 4	N
Anderson (1942)	299,796 $\pm$ 14	Y
Aslakson (1949)	299,792.3 $\pm$ 2.4	Y
Houston (1949)	299,795 $\pm$ 1	N
Bergstrand (1950)	299,792.7 $\pm$ 0.25	Y
Hansen and Bol (1950)	299,789.3 $\pm$ 0.4	N
Bearden and Watts (1951)	299,790.0 $\pm$ 0.9	N
Du Mond and Cohen (1951)	299,790.2 $\pm$ 0.9	N
Bergstrand (1952)	299,792 $\pm$ 6	Y

The current accepted value is: 299,792.4562 $\pm$ 0.0011

It is interesting to note that Michelson's measurement bounds still encompass the presently accepted value; he was one of the most gifted and careful experimenters of this century. Birge's value was a weighted estimate of contemporary data and serves to illustrate the hazards of such surveys.

The overall success here of .454 is fine for a major league baseball hitter but is sub par in the scientific arena. The added safety factor afforded by using the recommended Aerospace laboratory procedures is protection worth having, especially since the RMS approach, unless used with care and understanding will typically result in an underestimation of the error bounds.

A common sense approach, well laced with curiosity, suspicion and skepticism, is needed to ensure that an over-optimistic estimate of the error bounds does not occur.

REMEMBER: More students UNDERestimate than OVERestimate their errors.

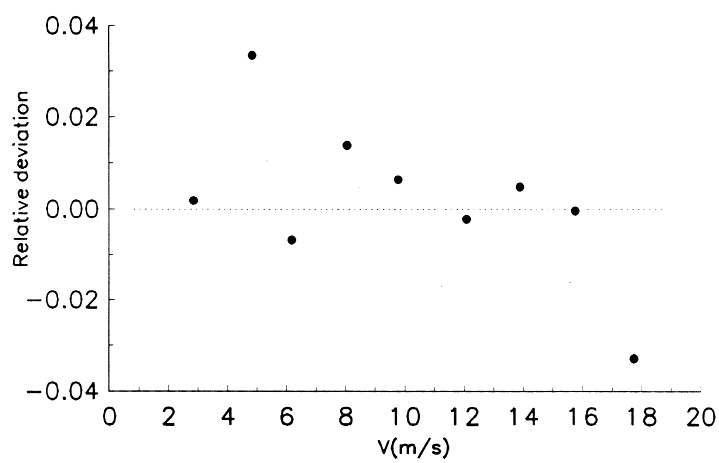
8.6 A Few More Examples

Treatment of "good" linear plots

If the experimental data fits a relation well then graphing the results in the usual manner (with error bars that disappear in the width of the drawn line) conveys little extra information over the tabulated data.

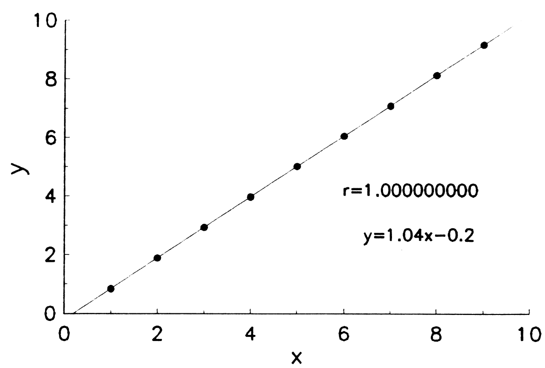
A more useful procedure is to plot the relative deviations. This is illustrated below by comparing the data from two velocity measuring instruments used in the LDV experiment. In this experiment the correlation coefficient is calculated by the computer but remember this ONLY indicates whether there is a linear relationship between the two variables, it does not disclose whether or not systematic deviations exist. The plot below gives one some confidence that there are no systematic deviations.

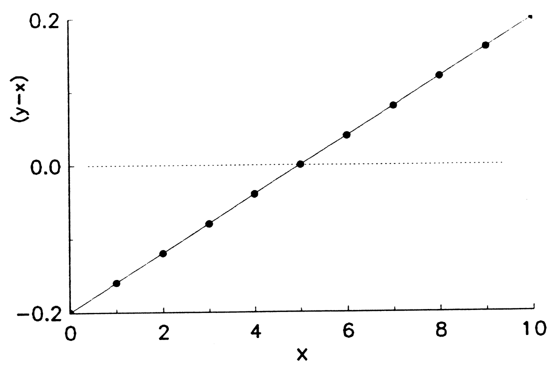
LDV	PITOT TUBE	RELATIVE DEVIATION
v_1 (m/s)	v_2 (m/s)	$\left(\frac{v_1 - v_2}{v_1}\right)$
2.84	2.835	.0018
4.82	4.659	.0034
6.17	6.212	-.0068
8.03	7.919	.0138
9.76	9.699	.0063
12.07	12.097	-.0022
13.87	13.064	.0048
15.73	15.736	-.0004
17.74	18.376	-.0329



The data below has been chosen to show that although two sets of measurements can be perfectly correlated there may be a systematic error between them which can be highlighted by a plot of the deviations. This systematic discrepancy may be due to heating, hysteresis, etc., but should alert the observer that something may be amiss and the experimental techniques deserve closer scrutiny.

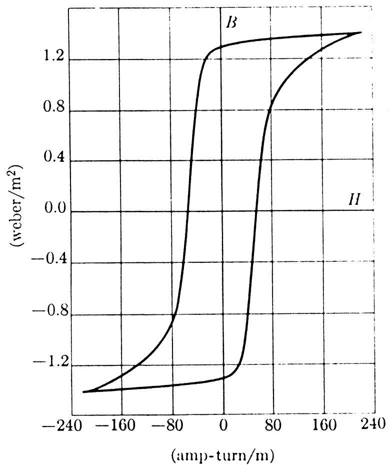
x	y	(y-x)
0	-.2	-.20
1	.84	-.16
2	1.88	-.12
3	2.92	-.08
4	3.96	-.04
5	5.00	0.00
6	6.04	+.04
7	7.08	+.08
8	8.12	+.12
9	9.12	+.16
10	10.2	+.20





Hysteresis

Most students are familiar with the classical magnetic hysteresis diagram below:



Hysteresis loop for iron.

However, many fail to note the far more prevalent occurrence of this phenomenon in numerous laboratory situations. There are even circumstances where this effect is present in Aerolab experiments.

Errors in a functional relation

Problem: Determine the error in the Mach number for the following expression:

$$\frac{p_1}{p_0} = \frac{(3.5M^2 - 0.5)^{2.5}}{29.508M^7} \rightarrow \ln p_1 - \ln p_0 = 2.5\ln{(3.5M^2 - 0.5)} - (ln29.508 + 7\ln M)$$

Differentiating:

$$\frac{dp_1}{p_1} - \frac{dp_0}{p_0} = \frac{2.5(7MdM)}{3.5M^2 - 0.5} - 7\frac{dM}{M}$$

Given: $p_1 = 202$, and $\Delta p_1 = 5$, $p_0 = 681$ and $\Delta p_0 = 5$, $M = 1.4887$

Substitution then gives:

$$|\frac{\Delta p_1}{p_1}| + |\frac{\Delta p_0}{p_0}| \approx |\frac{2.5(7M\Delta M)}{3.5M^2 - 0.5} - \frac{7\Delta M}{M}|$$

Therefore:

$$\Delta M = 0.02886$$

Integration errors

These include those associated with Simpson's rule, trapezoidal rule, square counting, etc. In some circumstances these can be treated analytically, but unfortunately for the student such cases are rare. More frequently there is no recourse other than to use common sense or carry computations through with upper and lower bounds included in the calculations.

Errors which occur in a differentiable function $f(x)$ integrated using the Trapezoidal rule are easy to treat, e.g:

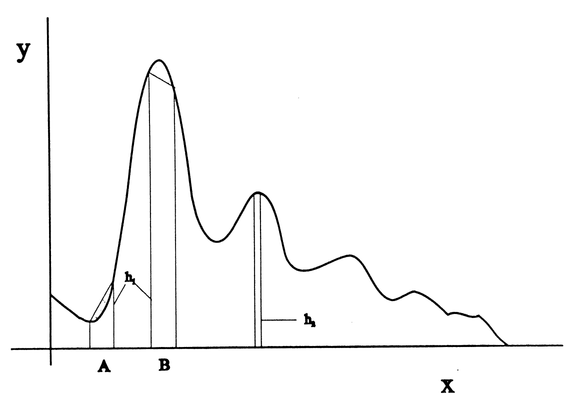
If $|\frac{\partial^2 f}{\partial x^2}| \leq M$ for all x in [a,b]

then the error is $\leq \frac{Mh^2(b-a)}{12}$

where the interval size is $h = \frac{b-a}{n}$

There are similar formulas for some of the other integration methods. If the functional relation is intractable then a simple approach, adequate for the Aerospace lab, is as follows:

Rough plot the curve to be integrated:



Select an interval width h which is sufficient to permit the slopes of the trapezoid to conform to the maximum curvature expected. In the figure above, h1 is adequate at x = A but is a little too coarse at x = B, and h2 would be a better overall choice.

The errors in the integration can be assessed by estimating the contributions of the shaded areas which are added or subtracted from the final integral. These areas tend to cancel when the integration covers equal concave and convex regions of the function.

Further insight can be gained from an evaluation of the uni-curved function $\frac{1}{x}$:

$$\int_1^2 \frac{dx}{x} = \ln 2 = 0.69314718...$$

Area integration of this function with an interval h = 0.1 gives the following results:

TRAPEZOIDAL = 0.69377
SIMPSON'S = 0.6931467

A coarser interval h = 0.5 gives:

TRAPEZOIDAL = 0.70833
SIMPSON'S = 0.69444

These results indicate the clear superiority of Simpson's rule but also show that providing the selected interval (h) is reasonably fine; a good approximation to the true integral can be made using less sophisticated methods.

9 Dimensional Analysis

We consider the technical system of units with the unit of force F, the unit of length L, and the unit of time T. The question of dimensional analysis is whether there exists a combination

$$V^\alpha a^\beta \rho^\gamma \mu^\delta$$

which is a pure number. This requires a determination of α, β, γ, and δ such that (quantities in square brackets means the dimension of that quantity):

$$\left[V^\alpha a^\beta \rho^\gamma \mu^\delta\right] = F^0 L^0 T^0 = [1]$$

Since, however, a dimensionless number raised to an arbitrary power remains a pure number, one of the quantities α, β, γ, δ is arbitrary. Putting therefore $\alpha = 1$ we get, substituting for the various physical quantities their dimensions,

$$\left[Va^{\beta}\rho^{\gamma}\mu^{\delta}\right] = \frac{LL^{\beta}F^{\gamma}T^{2\gamma}F^{\delta}T^{\delta}}{TL^{4\gamma}L^{2\delta}} = F^0L^0T^0$$

Equating the exponents of F, L, and T right and left, equations for β, γ and δ are obtained, namely,

$$\begin{aligned} \gamma + \delta &= 0 \\ 1 + \beta - 4\gamma - 2\delta &= 0 \\ 2\gamma + \delta - 1 &= 0 \end{aligned}$$

This leads to the solution

$$\beta = 1, \qquad \gamma = 1, \qquad \delta = 1$$

i.e., the only possible dimensionless combination of V, α, ρ, and μ is

$$Va\frac{\rho}{\mu} = R$$

If it had been known in advance that ρ and μ only appear in the combination μ/ρ, i.e., $\delta = -\gamma$ the derivation would have been still simpler. Since, therefore,

$$\left[\frac{\mu}{\rho}\right] = [\nu] = \frac{L^2}{T}$$

and

$$[Va] = \frac{L^2}{T}$$

it follows that Va/ν is the only possible combination giving a pure number.

Although this dimensional analysis physically is not so instructive as the similarity consideration, it has the advantage of being still applicable when the exact equation of motion is unknown and we know only which physical quantities are of importance for the phenomenon.

TABLE V

DIMENSIONS OF QUANTITIES DESCRIBING BOUNDARY, FLOW, AND FLUID CHARACTERISTICS

Quantity	Symbol	Dimensions in terms of	
		L-T-F	L-T-M
Geometric			
Length (any linear measurement)	L	L	L
Area	A	L^2	L^2
Volume	V	L^3	L^3
Slope	S		
Kinematic			
Time	t	T	T
Velocity, linear	v	L/T	L/T
angular	ω	$1/T$	$1/T$
Acceleration, linear	a	L/T^2	L/T^2
angular	α	$1/T^2$	$1/T^2$
Volume rate of flow, total	Q	L^3/T	L^3/T
per unit width	q	L^2/T	L^2/T
Circulation	Γ	L^2/T	L^2/T
Gravitational acceleration (γ/ρ)	g	L/T^2	L/T^2
Kinematic viscosity (μ/ρ)	ν	L^2/T	L^2/T
Dynamic			
Mass	M	FT^2/L	M
Force	F	F	ML/T^2
Mass density	ρ	FT^2/L^4	M/L^3
Specific weight	γ	F/L^3	M/L^2T^2
Dynamic viscosity	μ	FT/L^2	M/LT
Surface tension	σ	F/L	M/T^2
Elastic modulus	E	F/L^2	M/LT^2
Pressure intensity	p	F/L^2	M/LT^2
Shear intensity	τ	F/L^2	M/LT^2
Impulse, momentum	I, M	FT	ML/T
Work, energy	W, E	LF	ML^2/T^2
Power	P	LF/T	ML^2/T^3
Dimensionless			
Euler number	**E**		
Froude number	**F**		
Reynolds number	**R**		
Weber number	**W**		
Mach number	**M**		